**Satellite – Vessel Line of Sight Determination**

**A. Line of Sight Determination Assumption/General Considerations**

From the given scenario objective:

*“Determine the “hits” where a satellite has geodetic overlap of any vessel(s) at any point(s) in time. For simplicity, it may be assumed that a satellite has full view of half the earth (regardless of satellite type or its elevation above the earth). However, additional accuracy models with rationale is allowed.”*

Assuming any selected satellite has full view of (half) the earth’s sphere or a “hemisphere”, low earth-observing (LEO) satellites in retrograde orbits (e.g. including sun synchronous) will see all vessels – have line of sight (LOS) to all vessels – more than once every orbital period. The “horizon” for the satellite will be a “great circle”, since a great circle on the surface of the earth lies in a plane passing through the earth's center. On the other hand, prograde orbits with periods greater than 12 hours (e.g. Molniya) will not cover the earth every orbital period. If we assume that the LEO satellites only have a partial view of the earth’s surface (a more realistic assumption), then not all vessels will be seen by these satellites during every revolution, but only during some revolutions or not at all.

**B. Calculating the presence of a LOS for a given satellite and vessel at a given time**

The earth centered inertial (ECI) coordinate system, (a right-handed coordinate system, centered at the center of the earth, with z axis pointing north, and the x-y plane coincident with the earth’s equatorial plane with the x-axis pointing to the vernal equinox of epoch) is typically the common coordinate system used when performing coordinate satellite and earth-based observer transformations.

In the Space Track two-line element (TLE) data (the first data set example given in the problem statement), osculating (i.e., instantaneous) orbital elements are given for a selected satellite at a specific time, t0. For any given satellite these TLE reports are updated with new orbital elements reflecting new satellite observations at a frequency dependent on satellite type (e.g. most frequently for LEO satellites) designed to allow analytic propagators to maintain high accuracy. *Failure to make and report TLE updates w/r to LEO satellites can impact fidelity of the analysis.*

The specific TLE time, t0, is given in terms of “epoch day and Julian day fraction.” Also given in the data set is the “mean anomaly” and “mean motion” and mean motion rates of change of the satellite. From this information, the ECI-referenced position (and velocity) state vector of the satellite, r sat(t0) with rectangular components

[xsat(t0), y sat(t0), z sat(t0)]

at the specific time t0 may be obtained by the usual formulas of elliptic motion.

From the position state vector at time t0, the position state vector, rsat(tsat,i)of the satellite with rectangular components [xsat(tsat,i), y sat(tsat,i), z sat(tsat,i)] can be obtained at any earlier or later time, tsat,i, by analytical closed form orbital propagation models available that use TLE elements (e.g. SGP4) available in satellite toolkits.

The Automatic Identification System (AIS) (the second data set example given in the problem statement) gives the latitude and longitude of a vessel along with the specific time tepoch,vessel, vessel direction (vessel heading), vessel ground speed and vessel rate of turn. Assuming a spherical earth with constant radius, rearth, this data essentially gives spherical coordinates of the vessel and the rates of change of the spherical angles. *(A non-spherical earth could also be used if higher accuracy is desired.)* From the spherical coordinates of the vessel at time tepoch,vessel, and the rates of change of the spherical angles, the spherical coordinates of the vessel at a later or earlier time, ti,vessel, can be obtained, assuming no change in the vessel’s heading, speed and turn rate.

Here, it is assumed that we seek to test if there is a “hit” at ti,vessel = tsat,i.

From the spherical coordinates of the vessel at time ti,vessel the position state vector of the vessel, rvessel(ti,vessel) with rectangular components [xvessel(ti,vessel), yvessel(ti,vessel), zvessel(ti,vessel)] at the time tepoch,vessel may be obtained by the usual formulas between spherical and rectangular coordinates.

Since ti,vessel = tsat,i , let ti,vessel = tsat,i = ti. Now there are two vectors at time ti:

rsat(ti) = rsat[xsat(ti), y sat(ti), z sat(ti)]

and

rvessel(ti) = rvessel[xvessel(ti), yvessel(ti), zvessel(ti)]

The problem is to determine if the vessel is in the line of sight of the satellite (“geodetic overlap” as stated in the problem description/statement). The problem statement allows the assumption that a satellite has full view of half the earth. This would mean that if angle between the two vectors rsat(ti) and rvessel(ti) is less than 900, than the vessel is in the line of sight of the satellite and conversely if the angle between the two vectors is greater than 900, the vessel is not in the line of sight of the satellite. To determine the angle between two vectors, the “dot product” of two vectors can be used. For example, if the two vectors are A and B, the dot product of the two is given by

A dot B = |A| |B| cosine (angle between A and B)

Where |A| |B| indicates the product of the magnitudes of the vectors A and B – which of course is always positive. Using vector components the dot product can be written as:

A dot B = Ax Bx + Ay By + Az Bz = |A| |B| cosine (angle between A and B)

If the angle between the vectors is less than 900, then the cosine of the angle is positive. If the angle between the vectors is greater than 900, then the cosine of the angle is negative. So, if one forms A dot B, one can determine if the angle between the two is greater than or less than 900, by noting the sign of the product.

Using the satellite and vessel vectors:

If rsat[xsat(ti), y sat(ti), z sat(ti)] dot rvessel[xvessel(ti), yvessel(ti), zvessel(ti)] > 0, line of sight

If rsat[xsat(ti), y sat(ti), z sat(ti)] dot rvessel[xvessel(ti), yvessel(ti), zvessel(ti)] < 0, NO line of sight

The above test could be repeated over any other reported point in time for the selected vessel as well as across vessel to answer the question:

*“Determine the “hits” where a satellite has geodetic overlap of any vessel(s) at any point(s) in time”*

**C. Implementation Concepts**

The “determination of hits” problem has the following variants:

1. Given a satellite and a time, list all the vessels that are in its geodetic FOV;   OR

2.  Given a vessel and a time, list all the satellites whose geodetic FOV contains it;  OR

3.  Given a satellite and a time range, list all the windows (opening and closing) for all the vessels that will come into its geodetic FOV;  OR

4.  Given a vessel and a time range, list all the windows (opening and closing) for all the satellites that will have the vessel in their geodetic FOV.

The base solution to all the above is identical:  given a vessel, a satellite and a time:

1.  Calculate the vessel’s position, interpolating using dead reckoning between the two nearest (timewise) points

2.  Calculate the satellite’s position using the nearest (timewise) TLE and propagating forward or backward, as appropriate

3.  Convert both of the above to earth centered state vectors

4.  Calculate the angle between the vectors

5.  If the angle is less than the satellite’s FOV / 2 , then there is a hit.  NOTE: can assume 180-degree FOV, however calculation of the actual geodetic FOV from the satellite’s altitude is trivial

Given the base solution, each of the problems above can be solved as follows:

1.  Given a satellite and a time, run the base calculation against all the vessels in the database.

2.  Given a vessel and a time, run against all the satellites.

3.  Given a satellite and time range, run against all the vessels at every time point given a fixed interval (e.g. every minute)

4. Given a vessel and time range, run against all the satellites at every time point.

Assumptions:

1. Vessel positions will be interpolated

2.  Spherical earth assumption (vs reference ellipsoid)

3. Implementation for the base satellite vector calculation will use an existing orbital dynamics library (e.g. <https://pypi.org/project/sgp4/>).

4. Consider Trifacta to preprocess AIS data (time stamps, coordinate transformations)